Boundary Layer Phenomena and Perturbation Methods in Non-homogeneous Differential Equations

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Abstract

In this research, we discuss boundary layers in non-homogeneous second order differential equations of the form

$$\epsilon y'' + p(t)y' + q(t)y = f(t),\ 0 < t < 1,\ 0 < \epsilon << 1$$

$$y(0) = \alpha,\ y(1) = \beta,\ p(t) > 0,\ p(t) < 0.$$

Boundary layers occur in differential equations in which the perturbations are operative over very narrow regions across which the dependent variables undergo very rapid changes. These narrow regions frequently adjoin the boundaries of the domain of interest owing to the fact that the small parameter multiplies the highest derivative in the differential equation. Consequently, they are referred to as boundary layers in fluid mechanics, edge layers in solid mechanics, and skin layers in electrical applications. There are many physical situations in which the sharp changes occur inside the domain of interest, and the narrow regions across which these changes take place are usually referred to as shock layer in fluid and solid mechanics, transition points in quantum mechanics, and Stokes lines and surfaces in mathematics. These rapid changes cannot be handled by slow scales, but they can be handled by fast or magnitude or stretched scales.