Function Bounds for Solutions of Volterra Integro Dynamic Equations on Time Scales

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ABSTRACT

Introducing shift operators $\delta_{\pm}$ on time scales we construct the integro dynamic equation

$$x^{\Delta}(t) = -a(t)x(t) + \int_{t_0}^{t} b(\delta_{-}(s,t))x(s)\Delta s, \quad t \in [t_0, \infty)_{\mathbb{T}},$$

which includes the following Volterra equations in particular cases:

- *Volterra integro differential equation of convolution type*: For $\mathbb{T} = \mathbb{R}$ with $\delta_{-}(s,t) = t-s$ and $t_0 = 0$
$$x'(t) = -a(t)x(t) + \int_{0}^{t} b(t-s)x(s)ds, \quad t \in [0, \infty).$$
- *Volterra integral equation with fractional kernel:* For $\mathbb{T} = \mathbb{R}$ with $\delta_{-}(s,t) = t/s$ and $t_0 = 1$
$$x'(t) = -a(t)x(t) + \int_{1}^{t} b(\frac{t}{s})x(s)ds, \quad t \in [1, \infty).$$
- *Volterra integro difference equation of convolution type*: For $\mathbb{T} = \mathbb{Z}$ with $\delta_{-}(s,t) = t - s + \lambda$ and $t_0 = \lambda$
$$\Delta x(t) = -a(t)x(t) + \sum_{k=\lambda}^{t-1} b(t-k+\lambda)x(k), \quad t \in [\lambda, \infty) \cap \mathbb{Z}_{+},$$

Extending the scope of time scale variant of Gronwall's inequality we determine function bounds for the solutions of integro dynamic equation. Providing numerical examples we illustrate efficiency of obtained results. Moreover, we propose sufficient conditions implying exponential asymptotic stability of the trivial solution.